Probing the Fifth State of Matter with Relativistic Heavy Ions: A Theoretical Overview

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Abstract

I review the current status of lattice Monte-Carlo computations of the equation of state of QCD, our current understanding of the thermalization process at collider energies, and two new theoretical developments, one pertaining to the ab-initio calculation of the low-x gluon structure of heavy nuclei, the other to the chaotic behavior of QCD. At the end I give a brief overview of the status of various quark-gluon plasma signatures.

1 Introduction

In the preceding talk, Johanna Stachel [1] surveyed the status of relativistic heavy ion experiments with fixed targets at the Brookhaven AGS and the CERN-SPS. Whereas the observed hadron yields and spectra provide strong evidence that the final state—at the moment of break-up—is approximately thermal, we have little information about when the thermalization is achieved during the course of the nuclear reaction. Since the energy deposition processes at AGS and SPS energies are dominated by soft strong interactions, we currently also lack the theoretical tools to make reliable calculations. Hence we do not know yet, and may never know, whether a thermalized quark-gluon plasma is formed in these reactions.

The situation appears to be much more favorable at the higher energies where the future heavy ion colliders, RHIC and LHC, will operate. Two developments over the past few years have raised the hope that accurate calculations of the initial phase of nuclear reactions at such energies will eventually be possible. Firstly, numerical simulations of lattice QCD are rapidly approaching the stage where quantitatively reliable predictions of the equations of state of strongly interacting matter and its dynamical properties will be feasible. Secondly, the inclusion of color screening into parton cascades and nuclear structure functions opens up the prospect of a seamless description of the reaction dynamics within the framework of perturbative QCD from the initial nuclear structure up to a thermalized quark-gluon plasma. If this expectation comes to fruition, it will be possible to test quantitative theoretical predictions at RHIC and at the LHC.

In this talk I will review the current status of lattice Monte-Carlo computations of the equation of state of QCD, our current understand-

ing of the thermalization process at collider energies, and two recent new theoretical developments, one pertaining to the ab-initio calculation of the low-x gluon structure of heavy nuclei, the other to the chaotic behavior of QCD. At the end I will give a brief overview of the status of various quark-gluon plasma signatures.

2 The QCD Phase Diagram

Rigorous results about the phase diagram of QCD are based on Monte-Carlo simulations of lattice-QCD at finite temperature. For the pure SU(3)-gauge theory, quantitatively reliable results including an extrapolation to the continuum limit have recently become available. Calculations by the Bielefeld group [2] on lattices up to $32^3 \times 8$ yield a critical temperature (in units of the string tension σ) of

$$T_c/\sqrt{\sigma} = 0.629 \pm 0.003, \tag{1}$$

or $T_c = 260$ MeV if the value of σ derived from the Regge slope is used. The phase transition is of first order with a latent heat of slightly less than $2T_c^4$. The simulations provide clear evidence of significant interactions extending far above T_c. Fitting to predictions of thermal perturbation theory one finds, e.g.,

$$g^2(5T_c) \approx 1.5. \tag{2}$$

The simulations also provide information about quantities of dynamical relevance, such as the speed of sound, which is found rising from $c_s^2 < 0.1$ near T_c to $c_s^2 \approx 1/3$ at $T = 5T_c$.

Simulations of full lattice-QCD with light dynamical quarks have not yet reached this advanced stage, but have also made significant progress. State-of-the-art calculations [3] show a steep jump in the quantity ϵ/T^4 at $T_c \approx 150 - 200$ MeV, indicating a rapid unthawing of new degrees of freedom. The uncertainty in the value of T_c is mainly caused by discrepancies between different ways of setting the scale (string tension or hadron masses). Although it remains unclear whether thermodynamic quantities will show a singularity at T_c in the continuum limit and what order phase transition it may be, the simulations clearly demonstrate that the quark condensate $\langle\bar{\psi}\psi\rangle$ drops steeply at exactly the same temperature: color deconfinement and chiral symmetry restoration occur at the same temperature. The results are expected to reach a similar quality as those presently existing for the pure gauge theory when the next generation of parallel computers with close to teraflops performance will become available.

Perturbative approaches, valid far above T_c, have also made significant progress. The equation of state is now known [4,5] up to order g^5,

and a scheme yielding a complete perturbative result with only minimal lattice input has been worked out [6]. The "bad news" is that it is now understood that the convergence of the perturbative series (even in the sense of an asymptotic series) requires $g \leq 1$ or $\alpha_s(T) \leq 0.1$, which only holds for temperatures above the electroweak unification scale [7].

Nevertheless, thermal perturbation theory provides important insight into the dynamics of the high-temperature phase of QCD:

1. quarks and gluons develop dynamical masses of order gT.
2. Long-range color forces (except static color magnetism) are dynamically screened on distances of order $(gT)^{-1}$, providing an effective infrared cut-off for many transport coefficients.
3. Colored particles propagating through the plasma constantly exchange their color with the medium, rendering the quark-gluon plasma a poor color conductor. As will be discussed below, this "color chaos" facilitates the rapid equilibration of degrees of freedom in the plasma.

3 Initial conditions at RHIC and LHC

Most recent theoretical predictions for the initial conditions at which a thermalized quark-gluon plasma will be produced at heavy ion colliders are based on the concept of perturbative partonic cascades. The parton cascade model [8] starts from a relativistic transport equation of the form

$$p^\mu \frac{\partial}{\partial x^\mu} F_i(x,p) = C_i(x,p|F_k) \tag{3}$$

where $F_i(x,p)$ denote the phase space distributions of QCD quanta. The collision terms C_i are obtained in the framework of perturbative QCD from elementary $2 \rightarrow 2$ scattering amplitudes allowing for additional initial- and final state radiation due to scale evolution of the perturbative quanta. To regulate infrared divergences, the parton cascade model requires a cut-off for the $2 \rightarrow 2$ scattering (usually $p_T^{\min} = 1.5 - 2$ GeV$/c$) and a cut-off for time-like branchings ($\mu_0^2 = 0.5 - 1$ GeV$^2/c^2$).

Numerical simulations of such cascades for heavy nuclei provide a scenario where a dense plasma of (predominantly) gluons develops in the central rapidity region between the two colliding nuclei shortly after the impact. Detailed studies [9] show that the momentum spectrum of partons becomes isotropic and exponential, i.e. practically thermal, at a time $\tau \approx 0.7\Delta z$ in the rest frame of a slab of width Δz at central rapidity. To allow for a hydrodynamic description, the width of the slab should exceed the mean free path of a parton. Including color screening effects, one finds that the mean free path of a gluon in a thermalized plasma is $\lambda_f \approx (3\alpha_s T)^{-1}$ where T is the thermal slope of the parton

spectrum. For the very high initial values ($T \geq 0.7$ GeV) one concludes that a thermal hydrodynamic picture makes sense after $\tau_i \approx 0.3$ fm/c.

The high density of scattered partons in $A+A$ collisions makes it possible to replace the arbitrary infrared cut-off parameters $p_T^{\min}$ and μ_0^2 by dynamically calculated medium-induced cut-offs [10]. The dynamical density-dependent screening of color forces eliminates the need for $p_T^{\min}$, and the suppression of radiative processes provided by the Landau-Pomeranchuk-Migdal effect makes the virtuality cut-off μ_0^2 superfluous. Note that the viability of this concept crucially depends on the high parton density: the dynamical cut-off parameters must lie in the range of applicability of perturbative QCD. Since the density of initially scattered partons grows as $(A_1 A_2)^{1/3}(\ln s)^2$, this condition requires both large nuclei and high collision energy. The calculations indicate that this criterion will be met at RHIC and LHC but not at the presently accessible energies of the SPS and AGS. The framework is also not applicable to pp or $p\bar{p}$ collisions at current energies because the parton density remains too low.

The dynamic screening of parton cascades can be implemented as follows [11]. According to the uncertainty principle, a parton-parton collision can be considered as complete after a "formation time" $\tau_f \approx \hbar/p_T$ in the c.m. frame, where p_T is the momentum transfer. Accordingly, harder collisions are completed first. One can then consider the hard collisions as effectively screening the softer ones. The screening mass applicable to parton collisions of scale p_T is then determined as

$$\mu^2(p_T) = -\frac{3}{\pi^2}\alpha_s(p_T^2)\int_{p_T}^{\infty} d^3k |\nabla_k \, f(k)| \tag{4}$$

where $f(k)$ is the momentum density of more violently scattered partons. Feeding $\mu^2(p_T)$ back into the differential cross sections determining the number of scatterings one obtains a differential equation for $f(k)$ or, equivalently, $\mu^2(p_T)$. For Au + Au collisions at RHIC energy, the screening mass saturates at slightly below 1 GeV at small p_T, and at 1.5 GeV for Pb + Pb collisions at the LHC. Both these values are comfortably within the range of applicability of perturbative QCD, demonstrating that there may be no need for an artificial infrared cut-off. The screening of parton scattering by already scattered partons is analogous to the interaction among ladders in the traditional picture of soft hadronic interactions [12]. It would be interesting to rederive these results from this alternative point of view.

The self-consistent parton cascade makes parameter-free (though somewhat model dependent) predictions about the initial conditions achieved at RHIC and LHC. For the heaviest nuclei one expects thermalization to occur at $T = 730$ MeV (RHIC) or $T_c = 1150$ MeV (LHC) and

initial energy densities of about 60 GeV/fm^3 (RHIC) or 430 GeV/fm^3 (LHC). At the initial moment ($\tau \approx 0.25$ fm/c) the parton plasma is not chemically equilibrated. It takes another several fm/c to achieve chemical equilibrium. Following the evolution with the hydrodynamical model including self-consistent screening, the quark-gluon plasma is expected to last for about 5 fm/c until the temperature falls to T_c at RHIC and somewhat longer at the LHC. Final multiplicities are predicted to be $dN/dy \approx 1700$ (RHIC) and ≈ 8000 (LHC).

4 New Theoretical Approaches

4.1 Semiclassical parton structure

While representing a major advance in the application of perturbative QCD to relativisitic nuclear collisions, the self-screened parton cascade still relies on the input of experimentally measured parton structure functions of the colliding nuclei. A new approach, due to the Minneapolis group [13] promises to make these structure functions themselves calculable within perturbative QCD. The basic idea underlying this approach is that, as seen by partons with $x \leq 10^{-2}$, the valence quarks in a heavy nucleus constitute a very dense, sheet-like random color source. The large area density $\rho \approx 3A/\pi R^2$ defines a large scale parameter $\mu^2 = \rho$ at which the QCD coupling $\alpha_s(\mu^2)$ is weak. Introducing light-cone coordinates it is then possible to formulate a systematic program for calculating gluon and quark (sea) structure functions at small x as generated by classical random color fields and their quantum fluctuations. At the classical level, corresponding to the Weizsäcker-Williams approximation, the gluon density for not too small p_T is given by

$$\frac{1}{\pi R^2}\,\frac{dN}{dx dp_T^2} = \frac{\alpha_s \mu^2 (N_c^2 - 1)}{\pi^2 x p_T^2}. \tag{5}$$

Loop corrections will introduce $(\ln x)/x$ contributions, possibly leading to a power-like behavior at small x upon resummation.

The approach can be extended to the collision between two nuclei, viewed as the interaction among two counter-propagating sheets of valence quarks [14]. Since screening effects are already included in this approach, it may provide another description of the initial state produced in a nuclear collision at RHIC or LHC.

5 Chaotic Glue and Thermalization

A completely different approach to the thermalization problem in non-Abelian gauge theories emerges from solutions of the clasical Yang-Mills equations. Numerical calculations have shown that these equations form a strongly chaotic, infinite-dimensional dynamical system [15,16]. The evidence is derived from the study of the (properly defined) distance between two initially neighboring field configurations as a function of time. Neighboring configurations diverge at an exponential rate:

$$\|A_1^\mu(t) - A_2^\mu(t)\| \sim e^{\lambda t}. \tag{6}$$

The complete spectrum of Lyapunov exponents λ_i can be obtained from numerical solutions of the Yang-Mills equations on a lattice in Minkowski space. There is evidence that the lattice fields thermalize (in the microcanonical sense) and the Lyapunov exponents scale as (g^2T).

A fundamental result of nonlinear dynamics is that the sum of all positive Lyapunov exponents (the so-called Kolmogorov-Sinai entropy) measures the rate of entropy growth after coarse graining. The Lyapunov exponents, therefore, provide a measure of the thermal equilibration time. Using the largest Lyapunov exponent for the SU(3) gauge theory [17], one finds a characteristic time of order 0.2 - 0.3 fm/c in the range of temperatures (300-700 MeV) relevant for RHIC. This agrees nicely with the results from parton cascade simulations.

6 Quark-Gluon Plasma Signatures

A wide variety of signatures for the formation of a quark-gluon plasma in heavy-ion collisions has been proposed. Considerable progress has been made in recent years in understanding the background to many of these signals [18,19]. For example, it will be very difficult, if not impossible, to identify direct lepton pairs emitted by the plasma, because of a formidable background of decay leptons from D-mesons [20]. On the positive side, the suppression of charmonium states observed in $p + A$ collisions is becoming understood as due to the absorption of the color octet component in nuclei [21]. This provides a well-defined benchmark against which additional suppression effects can be identified. Indeed, the very high initial temperature now predicted for RHIC and the LHC will allow even charmed quarks to get into chemical equilibrium before hadronization. There is also a chance that even Υ mesons will be significantly suppressed at the LHC. The predicted production of multistrange baryons above thermal yields also remains a promising signature.

It has recently been understood that the energy loss of hard partons in a quark-gluon plasma may be much higher than expected [22]. It may even grow with the length of the traversed medium. Accordingly, the

quenching of jet production, or of leading high-p_T hadrons, has attracted increased interest as a probe of the early stage of the evolution of the plasma.

The formation of chirally disorientated domains of the quark condensate in the vacuum is a new, intriguing signature of the chiral phase transition [23]. Such domains correspond to coherent excitations of the pion field; they would decay by producing large fluctuations away from 1/3 in the π^0/π ratio. Detailed numerical studies [24] of the linear sigma model have shown how disorientated domains can grow if the chiral transition occurs rapidly, producing a temporarily unstable state of the chiral order parameter $\langle\bar{\psi}\psi\rangle$. Sophisticated techniques for the identification of domain structures, such as wavelet analysis [25], may help observing these domains if they are produced experimentally.

7 Summary

As RHIC proceeds toward completion, theoretical advances during the last few years are giving us a much clearer view of the physics to be expected in heavy ion collisions at that energy. Lattice-QCD simulations have unambiguously established the rapid "unthawing" of color and the restoration of chiral symmetry at a temperature below 200 MeV. Calculations of parton transport processes based on perturbative QCD predict very high initial temperatures, above 500 MeV at RHIC and 1 GeV at the LHC. New approaches to low-x parton structure and the thermalization problem hold the promise of a model and parameter-independent description of the formation and evolution of a quark-gluon plasma at RHIC energy and beyond. Finally, the various plasma signatures are becoming much better understood, providing valuable guidance for the experimental program at RHIC which will start in 1999.

Acknowledgments

This work was supported in part by a grant from the U.S. Department of Energy, Office of Energy Research (DE-FG02-96ER40945). The author also thanks the organizers of the PANIC-96 conference for support.

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